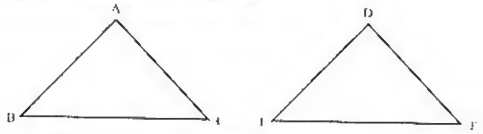
Unit 10

CONGRUENT TRIANGLES

Congruent Triangle



Let there be two triangles ABC and DEF. Out of the total six (1 – 1) correspondences that can be established between ΔABC and ΔDEF. One of the choices is explained below.

In the correspondence $\Delta ABC \leftrightarrow \Delta DEF$ it means.

$\angle A \leftrightarrow \angle D$ ($\angle A$ corresponds to $\angle D$)

$\angle B \leftrightarrow \angle E$ ($\angle B$ corresponds to $\angle E$)

$\angle C \leftrightarrow \angle F$ ($\angle C$ corresponds to $\angle F$)

$\overline{AB} \leftrightarrow \overline{DE}$ ($\overline{AB}$ corresponds to $\overline{DE}$)

$\overline{BC} \leftrightarrow \overline{EF}$ ($\overline{BC}$ corresponds to $\overline{EF}$)

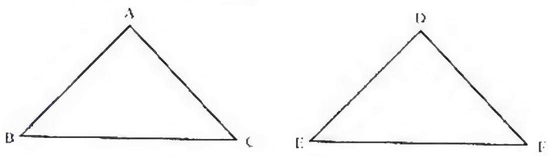
$\overline{CA} \leftrightarrow \overline{FD}$ ($\overline{CA}$ corresponds to $\overline{FD}$)

Congruency of Triangles

Two triangles are said to be congruent written symbolically as, $\cong$, if there exists a correspondence between them such that all the corresponding sides and angles are congruent i.e.

$$\text{If} \begin{cases} \overline{AB} \cong \overline{DE} \\ \overline{BC} \cong \overline{EF} \\ \overline{CA} \cong \overline{FD} \end{cases} \quad \text{and} \quad \begin{cases} \angle A \cong \angle D \\ \angle B \cong \angle E \\ \angle C \cong \angle F \end{cases}$$

Then $\Delta ABC \cong \Delta DEF$



Note

(i) These triangles are congruent w.r.t. the above mentioned choice of the (1 – 1) correspondence.

(ii) $\Delta ABC \cong \Delta ABC$

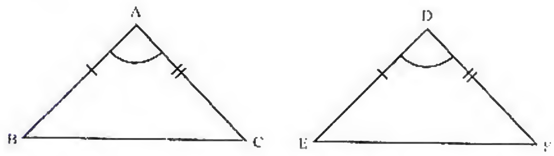
(iii) $\Delta ABC \cong \Delta DEF \Leftrightarrow \Delta DEF \cong \Delta ABC$

(iv) If $\Delta ABC \cong \Delta DEF$ and $\Delta ABC \cong \Delta PQR$, then $\Delta DEF \cong \Delta PQR$

In any correspondence of two triangles, if two sides and their included angle of one triangle are congruent to the corresponding two sides and their included angle of the other, then the triangles are congruent.

In $\Delta ABC \leftrightarrow \Delta DEF$, shown in the following figure.

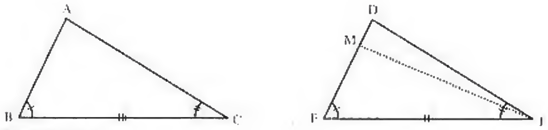
$$\text{If} \begin{cases} \overline{AB} \cong \overline{DE} \\ \angle A \cong \angle D \\ \overline{AC} \cong \overline{DF} \end{cases}$$



Then $\Delta ABC \cong \Delta DEF$ (S.A.S. Postulate)

Theorem

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the corresponding, side and angles of the other, then the triangles are congruent. (A.S.A $\cong$ A.S.A)



Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$

$\overline{BC} \cong \overline{EF}$

$\angle C \cong \angle F$

To prove

$\Delta ABC \leftrightarrow \Delta DEF$

Construction

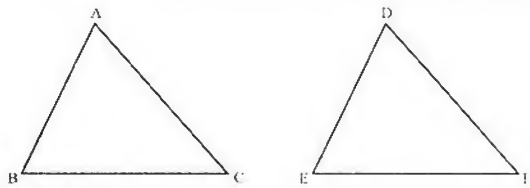
Suppose $\overline{AB} \not\cong \overline{DE}$, take a point M on $\overline{DE}$ such that $\overline{AB} \cong \overline{ME}$. Join M to F

Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{AB} \cong \overline{ME}$ ....(i)	Construction
$\overline{BC} \cong \overline{EF}$ ....(ii)	Given
$\angle B \cong \angle E$............(iii)	Given
$\therefore$ $\Delta ABC \cong \Delta MEF$	S.A.S. postulate
So, $\angle C \cong \angle MFE$	(Corresponding angles of congruent triangles)
But $\angle C \cong \angle DFE$	Given
$\therefore$ $\angle DFE \cong \angle MFE$	Both congruent to $\angle C$
This is possible only if D and M are the same points, and $\overline{ME} \cong \overline{DE}$	
So, $\overline{AB} \cong \overline{DE}$ ....(iv)	$\overline{AB} \cong \overline{ME}$ (construction) and
Thus from (i), (iii) and (iv), we have	$\overline{ME} \cong \overline{DE}$ (proved)
$\Delta ABC \cong \Delta DEF$	S.A.S. postulate

Example

In any correspondence of two triangles, if one side and any two angles of one triangle are congruent to the correspondence side and angles of the other, then the triangles are congruent. (S.A.A $\cong$ S.A.A.)



Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{BC} \cong \overline{EF}$, $\angle A \cong \angle D$, $\angle B \cong \angle E$

To Prove

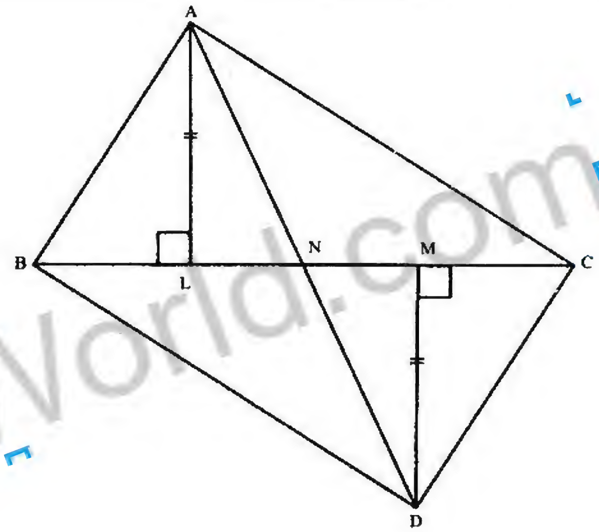
$\Delta ABC \cong \Delta DEF$

Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta DEF$	
$\angle B \cong \angle E$	Given
$\overline{BC} \cong \overline{EF}$	Given
$\angle C \cong \angle F$	$\angle A \cong \angle D$, $\angle B \cong \angle E$, (Given)
$\therefore$ $\Delta ABC \cong \Delta DEF$	A.S.A. $\cong$ A.S.A

Example

If ΔABC and ΔDCB are on the opposite sides of common base $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}$, $\overline{DM} \perp \overline{BC}$, $\overline{AL} \cong \overline{DM}$, then $\overline{BC}$ bisects $\overline{AD}$.



Given

ΔABC and ΔDCB are on the opposite sides of $\overline{BC}$ such that $\overline{AL} \perp \overline{BC}$, $\overline{DM} \perp \overline{BC}$, $\overline{AL} \cong \overline{DM}$ and $\overline{AD}$ is cut by $\overline{BC}$ at N.

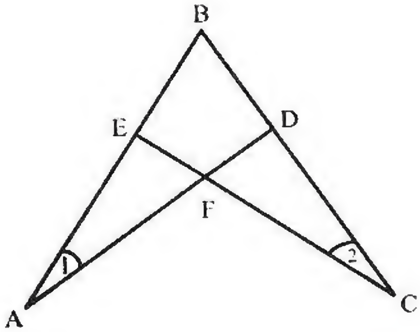
To Prove

$\overline{AN} \cong \overline{DN}$

Statements	Reasons
In $\Delta ALN \leftrightarrow \Delta DMN$	
$\overline{AL} \cong \overline{DM}$	Given
$\angle ALN \cong \angle DMN$	Each angle is right angle
$\angle ANL \cong \angle DNM$	Vertical angles
$\Delta ALN \cong \Delta DMN$	S.A.A. $\cong$ S.A.A
Hence $\overline{AN} \cong \overline{DN}$	Corresponding sides of $\cong$ Δs.

Exercise 10.1

1. In the given figure.
$\overline{AB} \cong \overline{CB}, \angle 1 \cong \angle 2.$
Prove that
$\Delta ABD \cong \Delta CBE$



Given

$\overline{AB} \cong \overline{CB}$

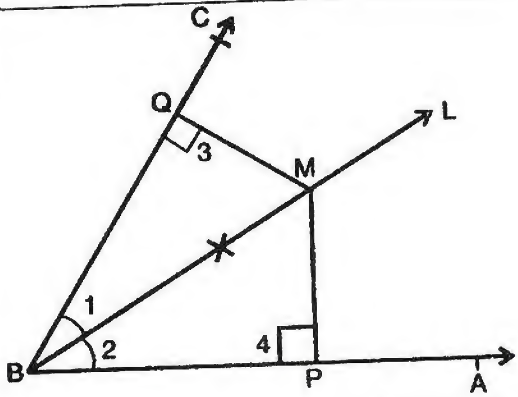
$\angle 1 = \angle 2$

To Prove

$\Delta ABD \cong \Delta CBE$

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBE$	
$\overline{AB} \cong \overline{CB}$	Given
$\angle 1 \cong \angle 2$	Given
$\angle ABD \cong \angle CBE$	Common angle
$\therefore$ $\Delta ABD \cong \Delta CBE$	A.S.A $\cong$ A.S.A

(2) From a point on the bisector of an angle, perpendiculars are drawn to the arms of the angle. Prove that these perpendiculars are equal in measure.



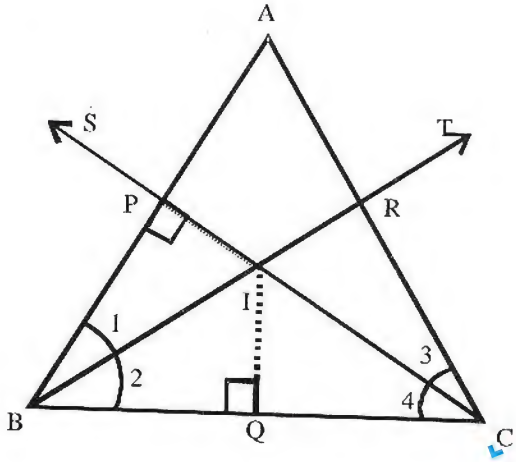
Given

$\angle ABC$, $\overrightarrow{BL}$ he bisector of $\angle ABC$, M any point on $\overrightarrow{BL}$, $\overline{MP}$ perpendicular on $\overline{AB}$, $\overline{MQ} \perp \overline{BC}$.

To Prove $\overline{MP} \cong \overline{MQ}$

Statements	Reasons
In $\Delta BMP \leftrightarrow \Delta BMQ$	
$\angle 1 \cong \angle 2$	$\overrightarrow{BL}$ bisects $\angle PBQ$
$\angle 3 \cong \angle 4$	Each = 90°
$\overline{BM} \cong \overline{BM}$	Common
$\Delta BMP \cong \Delta BMQ$	A.S.A $\cong$ A.S.A
$\overline{PM} \cong \overline{QM}$	Corresponding sides of the congruent triangles.

(3) In a triangle ABC, the bisectors of $\angle B$ and $\angle C$ meet in a point I. Prove that I is equidistant from the three sides of ΔABC.



Given

In ΔABC, $\overrightarrow{BT}, \overrightarrow{CS}$ are the bisectors of the angles B and C respectively.

To Prove

I is equidistant from the three sides of ΔABC i.e. $\overline{IP} \cong \overline{IQ} \cong \overline{IR}$

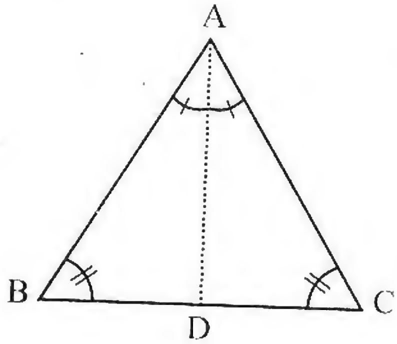
Construction

$\overline{IR} \perp \overline{AC}, \overline{IQ} \perp \overline{BC}, \overline{IP} \perp \overline{AB}$

Statements	Reasons
In $\Delta IPB \leftrightarrow \Delta IQB$	
$\angle 1 \cong \angle 2$	Given
$\angle P \cong \angle Q$	Each = 90°
$\overline{IB} \cong \overline{IB}$	Common
$\Delta IPB \cong \Delta IQB$	A.S.A ≅ A.S.A
$\overline{IP} \cong \overline{IQ}$ ....(i)	Corresponding sides of congruent triangles
Similarly $\Delta IRC \cong \Delta IQC$	
$\overline{IQ} \cong \overline{IR}$ ...(ii)	Corresponding sides of congruent triangles
$\overline{IP} \cong \overline{IQ} \cong \overline{IR}$	By (i) and (ii)

Theorem

If two angles of a triangle are congruent, then the sides opposite to them are also congruent.



Given

In ΔABC, $\angle B \cong \angle C$

To Prove

$\overline{AB} \cong \overline{AC}$

Construction

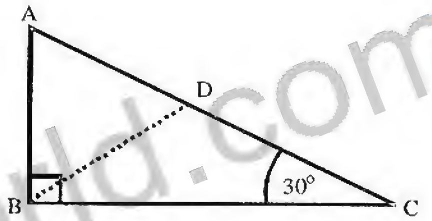
Draw the bisector of $\angle A$, meeting $\overline{BC}$ at the point D.

Proof

Statements	Reasons
In $\quad \Delta ABD \leftrightarrow \Delta ACD$	
$\overline{AD} \cong \overline{AD}$	Common
$\angle B \cong \angle C$	Given
$\angle BAD \cong \angle CAD$	Construction
$\therefore \quad \Delta ABD \cong \Delta ACD$	S.A.A. $\cong$ S.A.A.
Hence $\overline{AB} \cong \overline{AC}$	Corresponding sides of congruent triangles

Example

If one angle of a right triangle is of 30°, the hypotenuse is twice as long as the side opposite to the angle.



Given

In ΔABC, $m\angle B = 90^\circ$ and $m\angle C = 30^\circ$

To Prove

$m\overline{AC} = 2m\overline{AB}$

Construction

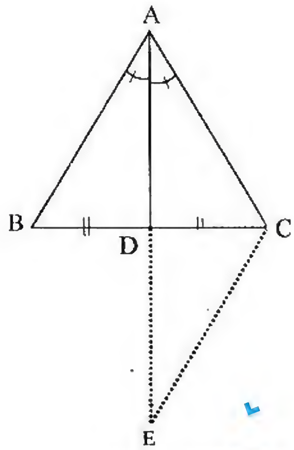
At B, construct $\angle CBD$ of 30°. Let $\overline{BD}$ cut $\overline{AC}$ at the point D.

Proof

Statements	Reasons
In $\quad \Delta ABD$, $m\angle A = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle C = 30^\circ$
$m\angle ABD = m\angle ABC - m\angle CBD = 60^\circ$	$m\angle ABC = 90^\circ$, $m\angle CBD = 30^\circ$
$\therefore \quad m\angle ADB = 60^\circ$	Sum of measures of $\angle$s of aΔ is 180°
$\therefore \quad \Delta ABD$ is equilateral	Each of its angles is equal to 60°
$\therefore \quad \overline{AB} \cong \overline{BD} \cong \overline{AD}$	Sides of equilateral Δ
InΔBCD, $\overline{BD} \cong \overline{CD}$	$\angle C = \angle CBD$ (each of 30°).
Thus $\left.\begin{aligned} m\overline{AC} &= m\overline{AD} + m\overline{CD} \\ &= m\overline{AB} + m\overline{AB} \\ &= 2(m\overline{AB}) \end{aligned}\right\}$	$\overline{AD} \cong \overline{AB}$ and $\overline{CD} \cong \overline{BD} \cong \overline{AB}$

Example

If the bisector of an angle of a triangle bisects the side opposite to it, the triangle is isosceles.



Given

In ΔABC, $\overline{AD}$ bisects $\angle A$ and $\overline{BD} \cong \overline{CD}$

To Prove

$\overline{AB} \cong \overline{AC}$

Construction

Produce $\overline{AD}$ to E, and take $\overline{ED} \cong \overline{AD}$.

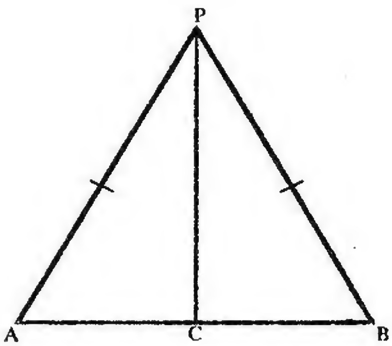
joint C to E

Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta EDC$	
$\overline{AD} \cong \overline{ED}$	Construction
$\angle ADB \cong \angle EDC$	Vertical angles
$\overline{BD} \cong \overline{CD}$	Given
$\therefore$ $\Delta ADB \cong \Delta EDC$	S.A.S. Postulate
$\therefore$ $\overline{AB} \cong \overline{EC}$ ....(1)	Corresponding sides of $\cong \Delta$s
and $\angle BAD \cong \angle E$	Corresponding angles of $\cong \Delta$s
But $\angle BAD \cong \angle CAD$	Given
$\therefore$ $\angle E \cong \angle CAD$	Each $\cong \angle BAD$
In ΔACE, $\overline{AC} \cong \overline{EC}$ ....(2)	$\angle E \cong \angle CAD$ (proved)
Hence $\overline{AB} \cong \overline{AC}$	From (1) and (2)

Exercise 10.2

Prove that a point, which is equidistant from the end points of a line segment, is on the right bisector of the line segment.



Given

$\overline{AB}$ is a line segment. Point P is such that $\overline{PA} \cong \overline{PB}$

To Prove

Point P is on the right bisector of $\overline{AB}$.

Construction

Join P to C, the midpoint of $\overline{AB}$

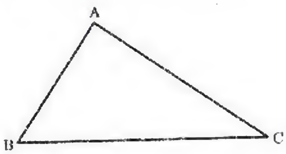
Proof

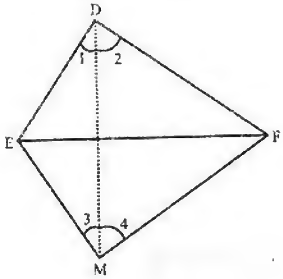
Statements	Reasons
In $\Delta ACP \leftrightarrow \Delta BCP$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PC} \cong \overline{PC}$	Common
$\overline{AC} \cong \overline{BC}$	Construction
$\Delta ACP \cong \Delta BCP$	$S.S.S \cong S.S.S$
$\angle ACP \cong \angle BCP$ ...(i)	Corresponding angles of congruent triangles
But $m\angle ACP + m\angle BCP = 180^o$...(ii)	supplementary angles,
$m\angle ACP = m\angle BCP = 90^o$	From (i) and (ii)
or $\overline{PC} \perp \overline{AB}$ ....(iii)	$m\angle ACP = 90^o$ (proved)
Also $\overline{CA} \cong \overline{CB}$ ....(iv)	construction
$\therefore$ $\overline{PC}$ is a right bisector Of $\overline{AB}$ i.e, the point P is on the right bisector of $\overline{AB}$.	from (iii) and (vi)

Theorem

In a correspondence of two triangles, if three sides of one triangle are congruent to the corresponding three sides of the other, then the two triangles are congruent.

($S.S.S. \cong S.S.S.$)





Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$ and $\overline{CA} \cong \overline{FD}$

To Prove

$\Delta ABC \cong \Delta DEF$

Construction

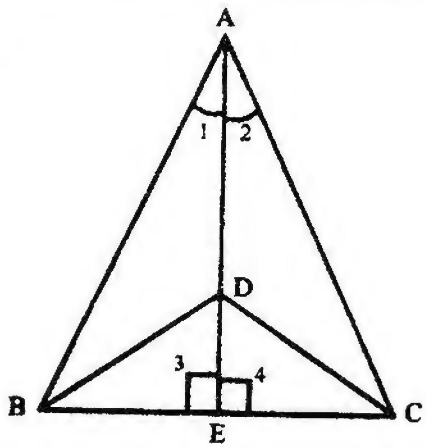
Suppose that in ΔDEF the side $\overline{EF}$ is not smaller than any of the remaining two sides. On $\overline{EF}$ construct a ΔMEF in which, $\angle FEM \cong \angle B$ and $\overline{ME} \cong \overline{AB}$. Join D and M. As shown in the above figures we label some of the angles as 1,2,3 and 4.

Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta MEF$	
$\overline{BC} \cong \overline{EF}$	Given
$\angle B \cong \angle FEM$	Construction
$\overline{AB} \cong \overline{ME}$	Construction
$\therefore$ $\Delta ABC \cong \Delta MEF$	S.A.S postulate
and $\overline{CA} \cong \overline{FM}$ ....(i)	(Corresponding sides of congruent triangles)
Also $\overline{CA} \cong \overline{FD}$ ....(ii)	Given
$\therefore$ $\overline{FM} \cong \overline{FD}$	From (i) and (ii)
In ΔFDM	
$\angle 2 \cong \angle 4$....................(iii)	$\overline{FM} \cong \overline{FD}$ (proved)
Similarly $\angle 1 \cong \angle 3$ ....(iv)	
$\therefore$ $m\angle 2 + m\angle 1 = m\angle 4 + m\angle 3$	{from (iii) and (iv)}
$\therefore$ $m\angle EDF = m\angle EMF$	
Now, In $\Delta DEF \leftrightarrow \Delta MEF$	
$\overline{FD} \cong \overline{FM}$	Proved
And $m\angle EDF \cong m\angle EMF$	Proved
$\overline{DE} \cong \overline{ME}$	Each one $\cong \overline{AB}$
$\therefore$ $\Delta DEF \cong \Delta MEF$	S.A.S postulate
Also $\Delta ABC \cong \Delta MEF$	Proved
Hence $\Delta ABC \cong \Delta DEF$	Each $\Delta \cong \Delta MEF$ (Proved)

Example

If two isosceles triangles are formed on the same side of their common base, the line through their vertices would be the right bisector of their common base.



Given

ΔABC and ΔDBC are formed on the same side of $\overline{BC}$ such that

$\overline{AB} \cong \overline{AC}$, $\overline{DB} \cong \overline{DC}$, $\overline{AD}$ meets $\overline{BC}$ at E.

To prove

$\overline{BE} \cong \overline{CE}$, $\overline{AE} \perp \overline{BC}$

Proof

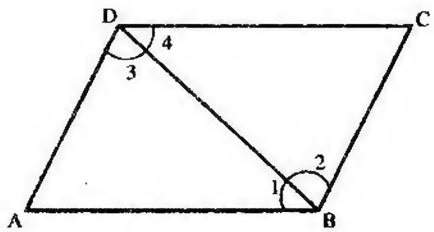
Statements	Reasons
In $\Delta ADB \leftrightarrow \Delta ADC$	
$\overline{AB} \cong \overline{AC}$	Given
$\overline{DB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{AD}$	Common
$\therefore$ $\Delta ADB \cong \Delta ADC$	S.S.S $\cong$ S.S.S.
$\therefore$ $\angle 1 \cong \angle 2$	Corresponding angles of $\cong$ Δs
In $\Delta ABE \leftrightarrow \Delta ACE$	
$\overline{AB} \cong \overline{AC}$	Given
$\angle 1 \cong \angle 2$	Proved
$\overline{AE} \cong \overline{AE}$	Common
$\therefore$ $\Delta ABE \cong \Delta ACE$	S.A.S. postulate
$\therefore$ $\overline{BE} \cong \overline{CE}$	Corresponding sides of $\cong$ Δs
$\angle 3 \cong \angle 4$ I	Corresponding angles of $\cong$ Δs
$m\angle 3 + m\angle 4 = 180^\circ$.........II	Supplementary angles Postulate
$\therefore$ $m\angle 3 = m\angle 4 = 90^\circ$	From I and II
Hence $\overline{AE} \perp \overline{BC}$	

Corollary: An equilateral triangle is an equiangular triangle.

Exercise 10.3

Q1. In the figure, $\overline{AB} \cong \overline{DC}$, $\overline{AD} \cong \overline{BC}$.

Prove that $\angle A \cong \angle C$, $\angle ABC \cong \angle ADC$.



Given $\overline{AB} \cong \overline{DC}$

$\overline{AD} \cong \overline{BC}$

To prove $\angle A \cong \angle C$

$\angle ABC \cong \angle ADC$

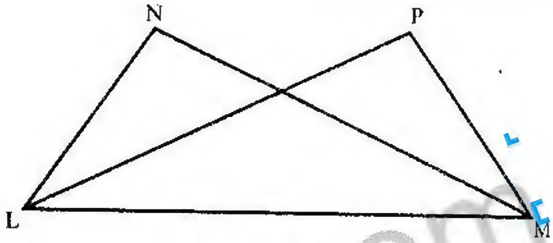
Proof

Statements	Reasons
In $\Delta ABD \leftrightarrow \Delta CBD$	
$\overline{AB} \cong \overline{DC}$	Given
$\overline{AD} \cong \overline{BC}$	Given

$\overline{BD} \cong \overline{BD}$	Common
$\therefore$ $\Delta ABD \cong \Delta CBD$	S.S.S $\cong$ S.S.S
$\angle A \cong \angle C$	
$\angle 1 \cong \angle 4$ ....(i)	Corresponding angles of congruent triangles
$\angle 2 \cong \angle 3$ ....(ii)	Corresponding angles of congruent triangles
$\angle 1 + \angle 2 = \angle 3 + \angle 4$	Adding (i) and (ii)
$\angle ABC \cong \angle ADC$	

2. In the figure, $\overline{LN} \cong \overline{MP}$, $\overline{MN} \cong \overline{LP}$.

Prove that $\angle N \cong \angle P, \angle NML \cong \angle PLM$.



Given

$\overline{LN} \cong \overline{MP}$

$\overline{LP} \cong \overline{MN}$

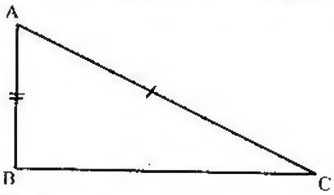
To prove

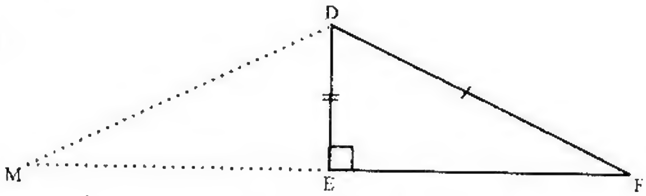
$\angle N \cong \angle P$, $\angle NML \cong \angle PLM$

Statements	Reasons
In $\Delta LMN \leftrightarrow \Delta LMP$	
$LM \cong \overline{MP}$	Given
$\overline{LP} \cong \overline{MN}$	Given
$\overline{LM} \cong LM$	Common
$\Delta LMN \cong \Delta LPM$	S.S.S $\cong$ S.S.S
$\angle N = \angle P$	Corresponding angles of congruent triangles
$\angle NML \cong \angle PLM$	Corresponding angles of congruent triangles

Theorem

If in the correspondence of the two right-angled triangles, the hypotenuse and one side of one triangle are congruent to the hypotenuse and the corresponding side of the other, then the triangles are congruent. (H.S $\cong$ H.S)





Given

In $\Delta ABC \leftrightarrow \Delta DEF$

$\angle B \cong \angle E$ (right angles)

$\overline{CA} \cong \overline{FD}$, $\overline{AB} \cong \overline{DE}$

To Prove $\Delta ABC \cong \Delta DEF$

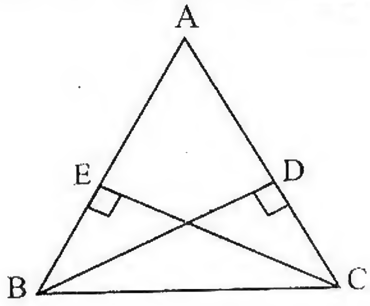
Construction Produce $\overline{FE}$ to a point M such that $\overline{EM} \cong \overline{BC}$ and join the points D and M.

Proof

Statements	Reasons
In $m\angle DEF + m\angle DEM = 180^\circ$...(i)	(Supplementary angles)
Now $m\angle DEF = 90^\circ$..................(ii)	(Given)
$\therefore$ $m\angle DEM = 90^\circ$	{from (i) and (ii)}
In $\Delta ABC \leftrightarrow \Delta DEM$	
$\overline{BC} \cong \overline{EM}$	(construction)
$\angle ABC \cong \angle DEM$	(each $\angle$ equal to 90°)
$\overline{AB} \cong \overline{DE}$	(given)
$\therefore$ $\Delta ABC \cong \Delta DEM$	S.A.S. postulate
And $\angle C \cong \angle M$	(Corresponding angles of congruent triangles)
$\overline{CA} \cong \overline{MD}$	(Corresponding sides of congruent triangles)
But $\overline{CA} \cong \overline{FD}$	(given)
$\therefore$ $\overline{MD} \cong \overline{FD}$	Each is congruent to $\overline{CA}$
In ΔDMF	
$\angle F \cong \angle M$	$\overline{FD} \cong \overline{MD}$ (Proved)
But $\angle C \cong \angle M$	(proved)
$\angle C \cong \angle F$	(each is congruent to $\angle M$)
In $\Delta ABC \leftrightarrow \Delta DEF$	
$\overline{AB} \cong \overline{DE}$	(given)
$\angle ABC \cong \angle DEF$	(given)
$\angle C \cong \angle F$	(proved)
Hence $\Delta ABC \cong \Delta DEF$	(S.A.A $\cong$ S.A.A)

Example

If perpendiculars from two vertices of a triangle to the opposite sides are congruent, then the triangle is isosceles.



Given

In ΔABC, $\overline{BD} \perp \overline{AC}$, $\overline{CE} \perp \overline{AB}$

Such that $\overline{BD} \cong \overline{CE}$

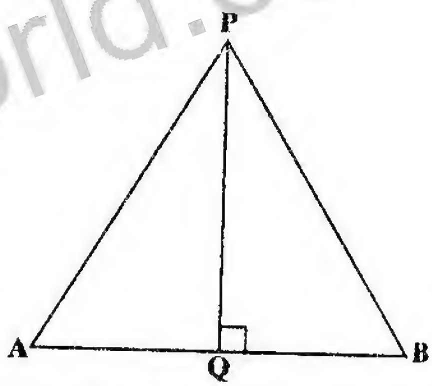
To Prove $\overline{AB} \cong \overline{AC}$

Proof

Statements	Reasons
In $\Delta BCD \leftrightarrow \Delta CBE$	
$\angle BDC \cong \angle BEC$	$\overline{BD} \perp \overline{AC}, \overline{CE} \perp \overline{AB}$ (given) $\Rightarrow$ each angle $= 90^\circ$
$\overline{BC} \cong \overline{BC}$	Common hypotenuse
$\overline{BD} \cong \overline{CE}$	Given
$\Delta BCD \cong \Delta CBE$	H.S. $\cong$ H.S.
$\angle BCD \cong \angle CBE$	Corresponding angles of $\cong$ Δs.
Thus $\angle BCA \cong \angle CBA$	
Hence $\overline{AB} \cong \overline{AC}$	In ΔABC, $\angle BCA \cong \angle CBA$

Exercise 10.4

1. In ΔPAB of figure, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$, prove that $\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$.



Given

In ΔPAB, $\overline{PQ} \perp \overline{AB}$ and $\overline{PA} \cong \overline{PB}$

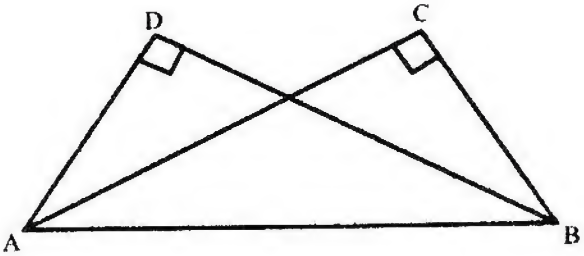
To Prove

$\overline{AQ} \cong \overline{BQ}$ and $\angle APQ \cong \angle BPQ$

Proof

Statements	Reasons
In $\Delta APQ \leftrightarrow \Delta BPQ$	
$\overline{PA} \cong \overline{PB}$	Given
$\overline{PQ} \cong \overline{PQ}$	Common
$\therefore$ $\Delta PAQ \cong \Delta PBQ$	H.S $\cong$ H.S
$\therefore$ $\overline{AQ} \cong \overline{BQ}$	Corresponding sides of congruent triangles
$\angle APQ \cong \angle BPQ$	Corresponding angles of the congruent triangles.

2. In the figure, $m\angle C = m\angle D = 90^\circ$ and $\overline{BC} \cong \overline{AD}$. Prove that $\overline{AC} \cong \overline{BD}$ and $\angle BAC \cong \angle ABD$.



Given

$m\angle C = m\angle D = 90^\circ$

$\overline{BC} \cong \overline{AD}$

To Prove

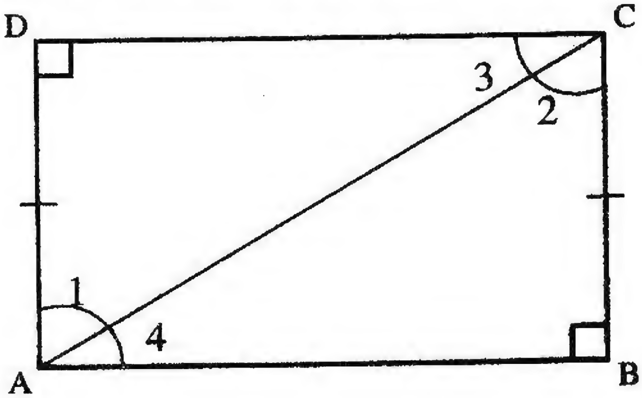
$\overline{AC} \cong \overline{BD}$

$\angle BAC \cong \angle ABD$

Proof

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta ABD$	
$m\angle C \cong m\angle D$	Each of 90°
$\overline{BC} \cong \overline{AD}$	Given
$\overline{AB} \cong \overline{AB}$	Common
	$H.S \cong H.S$
$\therefore$ $\Delta ABC \cong \Delta ABD$	
$\overline{AC} \cong \overline{BD}$	Corresponding sides of congruent triangles
$\therefore$ $\angle BAC \cong \angle ABD$	Corresponding angles of the congruent triangles

3. In the figure, $m\angle B = m\angle D = 90^\circ$ and $\overline{AD} \cong \overline{BC}$. Prove that ABCD is a rectangle.



Given

$m\angle B = m\angle D = 90^\circ$, $\overline{AD} \cong \overline{BC}$

Proof

ABCD is a rectangle

Statements	Reasons
In $\Delta ABC \leftrightarrow \Delta ADC$	
$m\angle B \cong m\angle D$	Each of 90°
$\overline{AD} \cong \overline{BC}$	Given
$\overline{AC} \cong \overline{AC}$	Common
$\therefore$ $\Delta ABC \cong \Delta ADC$	H.S $\cong$ H.S
$\overline{AB} \cong \overline{DC}$	
$\angle 1 \cong \angle 2$ ...(i)	
$\angle 4 \cong \angle 3$ ...(ii)	
$\angle 1 + \angle 4 = \angle 2 + m\angle 3$	
$\angle A = \angle C = 90^\circ$	
ABCD is a rectangle	By (i) and (ii)

4. Which of the following are true and which are false?

(i) A ray has two end points.
(ii) In a triangle, there can be only one right angle.
(iii) Three points are said to be collinear if they lie on same line.
(iv) Two parallel lines intersect at a point.
(v) Two lines can intersect only in one point.
(vi) A triangle of congruent sides has non-congruent angles.

Answers

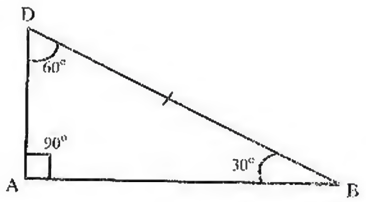
(i) False (ii) True (iii) True
(iv) False (v) True (vi) False

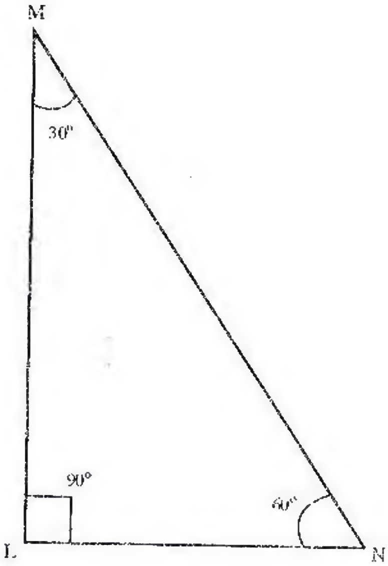
5. If $\Delta ABC \cong \Delta LMN$, then

(i) $m\angle M \cong$ ....
(ii) $m\angle N \cong$ ....
(iii) $m\angle A \cong$ ....

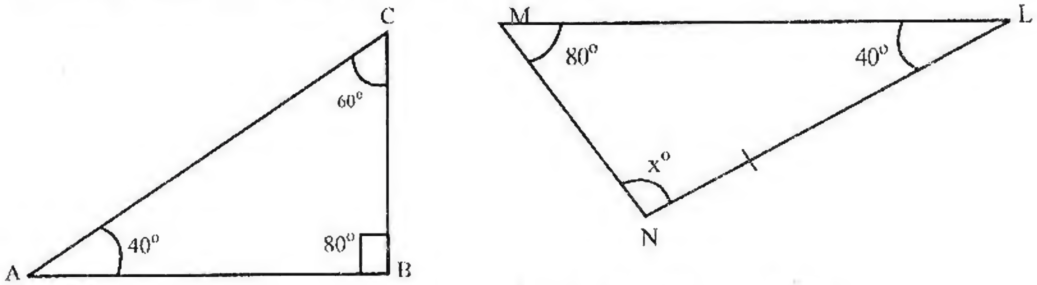
Answers

(i) $m\angle M \cong m\angle B$
(ii) $m\angle N \cong m\angle C$
(iii) $m\angle A \cong m\angle L$





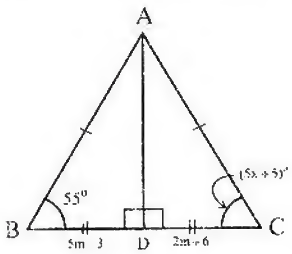
6. If $\Delta ABC \cong \Delta LMN$, then find the unknown x.



Answers

$x = 60^\circ$

7. Find the value of unknowns for the given congruent triangles.



$\Delta ABD \cong \Delta ACD$

$\overline{BD} \cong \overline{DC}$

$\Rightarrow \quad 5m - 3 = 2m + 6$

$5m - 2m = 3 + 6$

$3m = 9$

$m = \frac{9}{3} = 3$

Also

$\angle ACD \cong \angle ABD \Rightarrow$

Angles opposite to congruent sides are congruent

$5x + 5 = 55$

$5x = 55 - 5$

$5x = 50$

$x = \frac{50}{5}$

$x = 10$

8. If $\Delta PQR \cong \Delta ABC$, then find the unknowns.

$\Delta PQR \cong \Delta ABC$

$\overline{PQ} \cong \overline{AB}$

$x = 3$

$\overline{BC} \cong \overline{QR}$

$\Rightarrow z = 4$ cm

$\overline{AC} \cong \overline{PR}$

$y - 1 = 5$

$y = 5 + 1$

$y = 6$cm

$\therefore x = 3$cm, $y = 6$cm, $z = 4$cm